IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICAL METHODS
STANDARD LEVEL
PAPER 2

Wednesday 8 May 2002 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

*Please start each question on a new page. You are advised to show all working, where possible. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive **no** marks.*

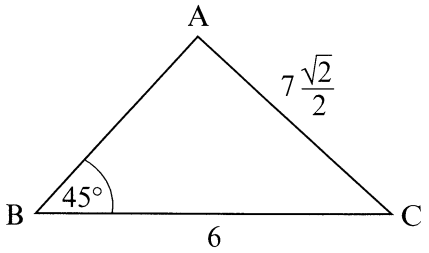
SECTION A

*Answer all **five** questions from this section.*

1. *[Maximum mark: 10]*

The diagram shows a triangle ABC in which $AC = 7\frac{\sqrt{2}}{2}$, $BC = 6$, $A\hat{B}C = 45°$.

Diagram not to scale



(a) Use the fact that $\sin 45° = \frac{\sqrt{2}}{2}$ to show that $\sin B\hat{A}C = \frac{6}{7}$. *[2 marks]*

The point D is on (AB), between A and B, such that $\sin B\hat{D}C = \frac{6}{7}$.

(b) (i) Write down the value of $B\hat{D}C + B\hat{A}C$.

(ii) Calculate the angle BCD.

(iii) Find the length of [BD]. *[6 marks]*

(c) Show that $\frac{\text{Area of } \Delta BDC}{\text{Area of } \Delta BAC} = \frac{BD}{BA}$. *[2 marks]*

2. *[Maximum mark: 11]*

Ashley and Billie are swimmers training for a competition.

(a) Ashley trains for 12 hours in the first week. She decides to increase the amount of time she spends training by 2 hours each week. Find the total number of hours she spends training during the first 15 weeks. *[3 marks]*

(b) Billie also trains for 12 hours in the first week. She decides to train for 10% longer each week than the previous week.

(i) Show that in the third week she trains for 14.52 hours.

(ii) Find the total number of hours she spends training during the first 15 weeks. *[4 marks]*

(c) In which week will the time Billie spends training first exceed 50 hours? *[4 marks]*

3. *[Maximum mark: 19]*

Three of the coordinates of the parallelogram STUV are $S(-2,-2)$, $T(7,7)$, $U(5,15)$.

(a) Find the vector $\overrightarrow{ST}$ and hence the coordinates of V. *[5 marks]*

(b) Find a vector equation of the line (UV) in the form $\boldsymbol{r} = \boldsymbol{p} + \lambda\boldsymbol{d}$ where $\lambda \in \mathbb{R}$. *[2 marks]*

(c) Show that the point E with position vector $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$ is on the line (UV), and find the value of λ for this point. *[2 marks]*

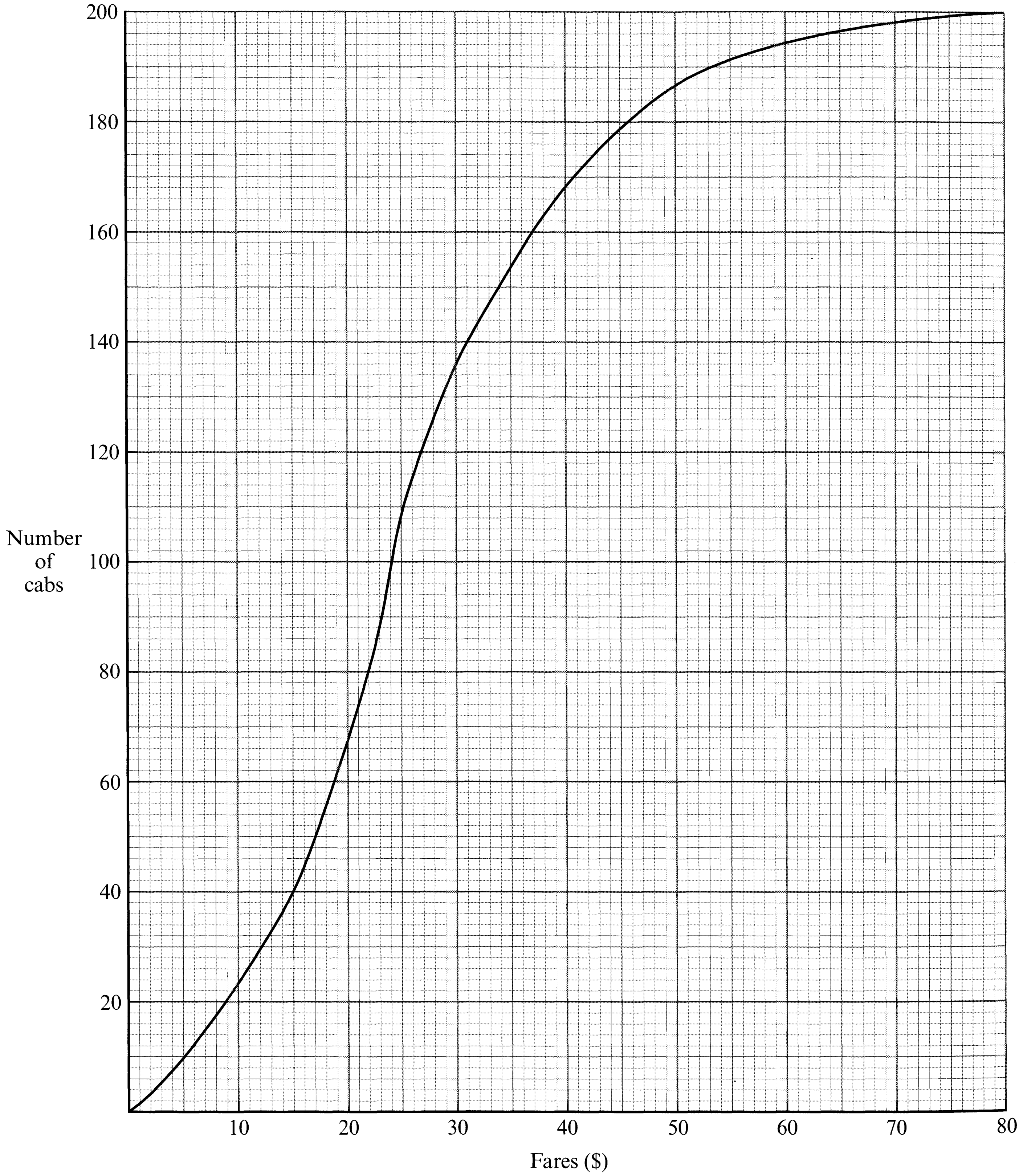
The point W has position vector $\begin{pmatrix} a \\ 17 \end{pmatrix}$, $a \in \mathbb{R}$.

(d) (i) If $|\overrightarrow{EW}| = 2\sqrt{13}$, show that one value of a is -3 and find the other possible value of a.

(ii) For $a = -3$, calculate the angle between $\overrightarrow{EW}$ and $\overrightarrow{ET}$. *[10 marks]*

4. *[Maximum mark: 17]*

(i) A taxi company has 200 taxi cabs. The cumulative frequency curve below shows the fares in dollars ($) taken by the cabs on a particular morning.



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(Question 4(i) continued)

(a) Use the curve to estimate

(i) the median fare;

(ii) the number of cabs in which the fare taken is \$35 or less. *[2 marks]*

The company charges 55 cents per kilometre for distance travelled. There are no other charges. Use the curve to answer the following.

(b) On that morning, 40% of the cabs travel less than a km. Find the value of a. *[4 marks]*

(c) What percentage of the cabs travel more than 90 km on that morning? *[4 marks]*

(ii) Two fair dice are thrown and the number showing on each is noted. The sum of these two numbers is S. Find the probability that

(a) S is less than 8; *[2 marks]*

(b) at least one die shows a 3; *[2 marks]*

(c) at least one die shows a 3, given that S is less than 8. *[3 marks]*

5. *[Maximum mark: 13]*

Consider functions of the form $y = \mathrm{e}^{-kx}$.

(a) Show that $\int_0^1 \mathrm{e}^{-kx}\,\mathrm{d}x = \frac{1}{k}(1-\mathrm{e}^{-k})$. *[3 marks]*

(b) Let $k = 0.5$

(i) Sketch the graph of $y = \mathrm{e}^{-0.5x}$, for $-1 \le x \le 3$, indicating the coordinates of the y-intercept.

(ii) Shade the region enclosed by this graph, the x-axis, y-axis and the line $x = 1$.

(iii) Find the area of this region. *[5 marks]*

(c) (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of k, where $y = \mathrm{e}^{-kx}$.

The point P(1, 0.8) lies on the graph of the function $y = \mathrm{e}^{-kx}$.

(ii) Find the value of k in this case.

(iii) Find the gradient of the tangent to the curve at P. *[5 marks]*

SECTION B

*Answer **one** question from this section.*

Statistical Methods

6. *[Maximum mark: 30]*

(i) The mass of packets of a breakfast cereal is normally distributed with a mean of 750 g and standard deviation of 25 g.

(a) Find the probability that a packet chosen at random has mass

(i) less than 740 g ;

(ii) at least 780 g ;

(iii) between 740 g and 780 g . *[5 marks]*

(b) Two packets are chosen at random. What is the probability that both packets have a mass which is less than 740 g ? *[2 marks]*

(c) The mass of 70% of the packets is more than x grams. Find the value of x . *[2 marks]*

(ii) Three schools from the same city enter students for an examination in which successful candidates can achieve one of three grades: *Pass*, *Credit*, *Distinction*. The results are shown in the following table

	Pass	*Credit*	*Distinction*
School A	8	22	40
School B	12	45	53
School C	11	29	20

It may be assumed that a student's result is independent of the school attended.

(a) The following table gives the expected frequencies for the above data.

	Pass	*Credit*	*Distinction*
School A	a	b	33.0
School B	c	d	51.8
School C	7.75	24.0	28.3

(i) Calculate the values a , b , c , d .

(ii) Find χ^2 for this data. *[7 marks]*

(This question continues on the following page)

(Question 6 (ii) continued)

(b) Newspapers wish to use these results to make comparisons between the schools. Based on the value of χ^2, decide whether there is justification for the statement 'success in the examination depends on which school is attended'. Examine the statement

(i) at the 5% level of significance;

(ii) at the 10% level of significance. *[4 marks]*

(iii) A scientist is investigating the way in which the length of a metal rod varies with temperature. She reads the length y mm at different temperatures x °C. From a set of these readings, she calculates the following results.

$\bar{x} = 200$, $\bar{y} = 1000$, $s_x = 2.31$, $s_y = 11.7$, $s_{xy} = 26.1$

(a) Find

(i) the product-moment correlation coefficient r;

(ii) the equation of the regression line of y on x;

(iii) the length of the rod when the temperature is 170°C. *[8 marks]*

(b) Which of the following diagrams most closely resembles the set of readings taken by the scientist? Give a reason for your answer. *[2 marks]*

Diagram A

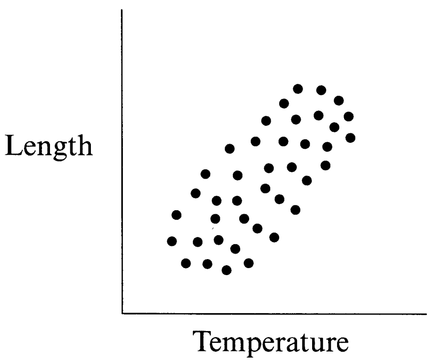


Diagram B

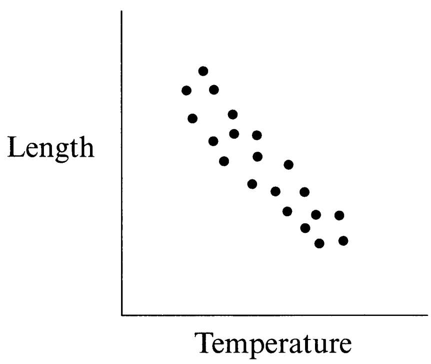


Diagram C

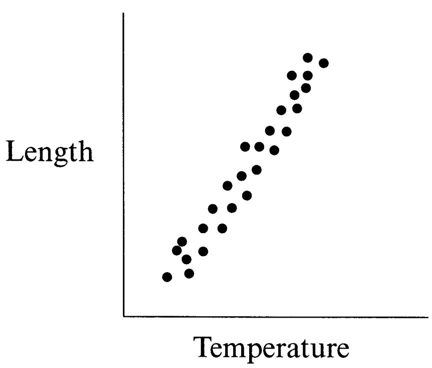
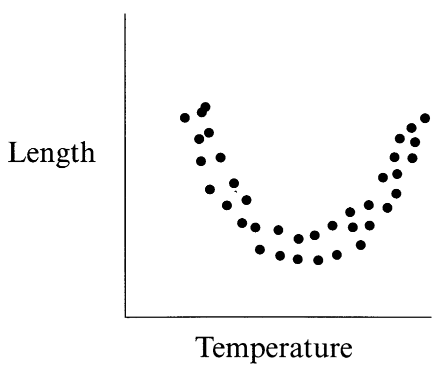


Diagram D



Further Calculus

7. *[Maximum mark: 30]*

(i) (a) Using the substitution $u = \cos x$ or otherwise, find $\int \sin^3 x \mathrm{d}x$. *[3 marks]*

(b) **Hence** find the area between the graph of $y = \sin^3 x$ and the x-axis, between $x = 0$ and $x = \frac{\pi}{2}$. *[2 marks]*

(ii) Let the function f be defined by $f(x) = \frac{2}{1 + x^3}$, $x \neq -1$.

(a) (i) Write down the equation of the vertical asymptote of the graph of f.

(ii) Write down the equation of the horizontal asymptote of the graph of f.

(iii) Sketch the graph of f in the domain $-3 \leq x \leq 3$. *[4 marks]*

(b) (i) Using the fact that $f'(x) = \frac{-6x^2}{(1 + x^3)^2}$, show that the second derivative $f''(x) = \frac{12x(2x^3 - 1)}{(1 + x^3)^3}$.

(ii) Find the x-coordinates of the points of inflexion of the graph of f. *[6 marks]*

(c) The table below gives some values of $f(x)$ and $2f(x)$.

x	$f(x)$	$2f(x)$
1	1	2
1.4	0.534188	1.068376
1.8	0.292740	0.585480
2.2	0.171703	0.343407
2.6	0.107666	0.215332
3	0.071429	0.142857

(i) Use the trapezium rule with five sub-intervals to approximate the integral $\int_1^3 f(x)\mathrm{d}x$.

(ii) Given that $\int_1^3 f(x)\mathrm{d}x = 0.637599$, use a diagram to explain why your answer is greater than this. *[5 marks]*

(This question continues on the following page)

(Question 7 continued)

(iii) Let $h(x) = \ln 2x - \sin\left(\frac{1}{2}x\right)$, $x > 0$.

(a) Show that $h(x) = 0$ has a root between 0.5 and 1 . *[3 marks]*

(b) The equation $h(x) = 0$ can be written in the form $x = \frac{1}{2}e^{\sin\left(\frac{1}{2}x\right)}$.

Let $g(x) = \frac{1}{2}e^{\sin\left(\frac{1}{2}x\right)}$

(i) Using the iteration formula $x_{n+1} = g(x_n)$ with starting value $x_0 = 1$

(a) write down x_1 and x_2;

(b) find the solution to $h(x) = 0$, correct to **six** significant figures.

(ii) Find $g'(x)$ and hence show that for any starting value x_0, the equation $x_{n+1} = g(x_n)$ will always give the root of $h(x) = 0$. *[7 marks]*

Further Geometry

8. *[Maximum mark: 30]*

(i) (a) $\boldsymbol{R}_1$, $\boldsymbol{R}_2$ are anti-clockwise rotations of 45° and 60° about the origin. Write down the matrices $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, using **exact** values. *[4 marks]*

(b) (i) What transformation is represented by the composition $\boldsymbol{R}_1\boldsymbol{R}_2$?

(ii) Hence show that $\cos 105° = \dfrac{1-\sqrt{3}}{2\sqrt{2}}$. *[5 marks]*

(ii) A linear transformation $\boldsymbol{P}$ maps the point $(1, 0)$ to $(1, 6)$ and the point $(0, 1)$ to $(3, 4)$.

(a) Write down the matrix $\boldsymbol{P}$. *[1 mark]*

(b) Find the image of each of the points $(1, 2)$ and $(-1, 1)$ under $\boldsymbol{P}$. *[3 marks]*

(c) Hence, or otherwise, find the equations of the two lines which are invariant under $\boldsymbol{P}$. *[4 marks]*

(d) Find the area scale factor for the transformation $\boldsymbol{P}$. *[2 marks]*

(iii) A transformation $\boldsymbol{T}$, representing reflection in a line l, is described by

$$\boldsymbol{T}: \begin{pmatrix} x \\ y \end{pmatrix} \to \frac{1}{17}\begin{pmatrix} -15 & 8 \\ 8 & 15 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 8 \\ -2 \end{pmatrix}.$$

(a) Find the image of each of the following points under $\boldsymbol{T}$

(i) $(0, 0)$;

(ii) $(8, -2)$;

(iii) $(2, -9)$. *[5 marks]*

(b) Show the points in part (a) and their images on a diagram.

(c) Hence, or otherwise, find the equation of l in the form $ax + by + c = 0$, where a, b, $c \in \mathbb{Z}$. *[6 marks]*